Theorem: If x is an even integer then x^2 is an even integer

Proof: Let x be an even integer. So $x = 2k$

then $x^2 = 4k^2$ $\quad x^2 = 2kx$

$x^2 = 2(2k^2)$ $\quad x^2 = 2(kx)$

Want $x^2 = 2$(some integer) so x^2 is even.

Def: $x \in \mathbb{Z}$ is even if $\exists y \in \mathbb{Z}$ s.t. $x = 2y$.

$2 \mid x$

Proposition: If x is odd then x^2 is odd.

If x is odd then $x = 2k+1$ for some k.

$\therefore x^2 = (2k+1)^2 = 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$

so $x^2 = 2t + 1$ and thus x^2 is odd.

Prop: Suppose $x \in \mathbb{Z}$, and x^2 is even.

Show x is even

$(P \Rightarrow Q) \Longleftrightarrow (\neg Q \Rightarrow \neg P)$

If x is NOT even then x^2 is not even.

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x^2 even so $x^2 = 2K$ for some K.